Pearson

Mark Scheme (Results)

Summer 2017

Pearson Edexcel International A Level
in Further Pure Mathematics F2
(WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where}\ |pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where}\ |pq| = |c|\ \text{and}\ |mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

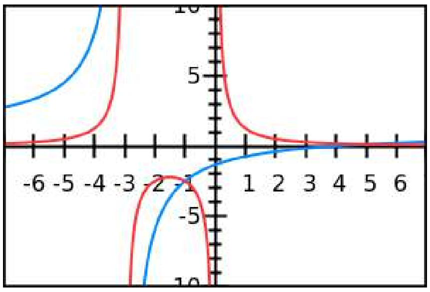
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Notes	Marks
1	$2(\cos 0+\mathrm{i}\sin 0)$ or 2	$(z=)\ 2$ or $(z=)\ 2(\cos 0+\mathrm{i}\sin 0)$ or $2\cos 0+\mathrm{i}\sin 0$ or $2+0\mathrm{i}$ Allow $2(\cos 0\pi+\mathrm{i}\sin 0\pi)$	B1
	$2\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$	This answer in this form. Do not allow e.g. $2\mathrm{e}^{\frac{2\pi}{5}\mathrm{i}}$ but allow $2\cos\frac{2\pi}{5}+2\mathrm{i}\sin\frac{2\pi}{5}$	B1
	$2\left(\cos\frac{2k\pi}{5}+\mathrm{i}\sin\frac{2k\pi}{5}\right),(k=2,3,4)$	Attempts at least 2 more solutions whose arguments differ by $\frac{2\pi}{5}$. Allow this mark if the arguments are out of range. May be implied by their answers.	M1
	Note that this answer in general solution form can score full marks if correct i.e. the A marks below can be implied. E.g. $z=2\left(\cos\frac{2k\pi}{5}+\mathrm{i}\sin\frac{2k\pi}{5}\right),(k=0,\ 1,\ 2,\ 3,\ 4)$ scores full marks		
	$2\left(\cos\frac{4\pi}{5}+\mathrm{i}\sin\frac{4\pi}{5}\right)$ $2\left(\cos\frac{6\pi}{5}+\mathrm{i}\sin\frac{6\pi}{5}\right)$ $2\left(\cos\frac{8\pi}{5}+\mathrm{i}\sin\frac{8\pi}{5}\right)$	A1: One further correct answer, allow the brackets to be expanded. A1: All correct, allow the brackets to be expanded.	A1 A1
Do not allow $2\left(\cos\frac{4\pi}{5}-\mathrm{i}\sin\frac{4\pi}{5}\right)$ or $2\left(\cos\left(-\frac{4\pi}{5}\right)+\mathrm{i}\sin\left(-\frac{4\pi}{5}\right)\right)$ for $2\left(\cos\frac{6\pi}{5}+\mathrm{i}\sin\frac{6\pi}{5}\right)$ Do not allow $2\left(\cos\frac{2\pi}{5}-\mathrm{i}\sin\frac{2\pi}{5}\right)$ or $2\left(\cos\left(-\frac{2\pi}{5}\right)+\mathrm{i}\sin\left(-\frac{2\pi}{5}\right)\right)$ for $2\left(\cos\frac{8\pi}{5}+\mathrm{i}\sin\frac{8\pi}{5}\right)$			
Ignore answers outside the range. For a fully correct solution that has extra solutions in range, deduct the final A mark.			
Answers in **degrees**: Penalise once the first time it occurs. Answers in degrees are: 0, 72, 144, 216, 288			
			(5)
			Total 5

Question Number	Scheme	Notes	Marks
2.	$\frac{x-4}{(x+3)} \le \frac{5}{x(x+3)}$		
Way 1	$\frac{x-4}{(x+3)} - \frac{5}{x(x+3)} (\le 0)$	Collects expressions to one side	M1
	$\frac{x^2-4x-5}{x(x+3)} (\le 0)$	M1: Attempt common denominator A1: Correct single fraction	M1A1
	$x = 0, -3$	Correct critical values	B1
	$x^2-4x-5 \Rightarrow (x-5)(x+1) = 0$ $\Rightarrow x = \ldots$	Attempt to solve their quadratic as far as $x = \ldots$ to obtain the **other** 2 critical values	M1
	$x = -1, 5$	Correct critical values	A1
	$-3 < x \le -1,\ 0 < x \le 5$ or e.g. $(-3, -1] \cup (0, 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b$, $c * x * d$ where * is "<" or "≤" and $a < b < c < d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	dM1A1A1
Notes Intervals may be separated by commas, written separately, $\cup$ or "or" or "and" may be used but not $\cap$ All marks are available for correct work if "=" is used instead of "$\le$" for the first 6 marks			
			(9)
			Total 9
Way 2	Multiplies by $x^2(x+3)^2$		
	$x^2(x+3)(x-4) \le 5x(x+3)$	Multiplies both sides by $x^2(x+3)^2$. May multiply by more terms but must be a positive multiplier containing $x^2(x+3)^2$	M1
	$x^3(x+3) - 4x^2(x+3) - 5x(x+3) \le 0$	M1: Collects expressions to one side A1: Correct inequality (or equation)	M1A1
	$x = 0, -3$	Correct critical values	B1
	$x(x+3)(x-5)(x+1) = 0 \Rightarrow x = \ldots$	Attempt to solve their quartic as far as $x = \ldots$ to obtain the **other** 2 critical values. Allow the x and $x + 3$ to be "cancelled" to obtain the other critical values so may end up solving a cubic or even a quadratic.	M1
	$x = -1, 5$	Correct critical values	A1
	$-3 < x \le -1,\ 0 < x \le 5$ or e.g. $(-3, -1] \cup (0, 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b$, $c * x * d$ where * is "<" or "≤" and $a < b < c < d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	dM1A1A1
			(9)

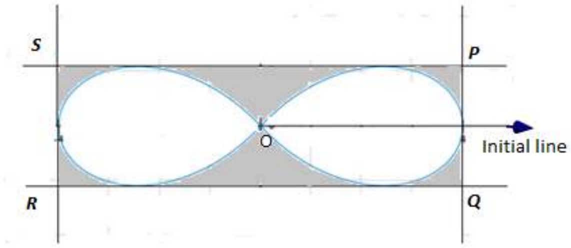
		Draws a sketch of graphs $y=\frac{x-4}{x+3}$ and $y=\frac{5}{x\left(x+3\right)}$	
Way 3	$x=0,-3$	Correct critical values (vertical asymptotes)	B1
	$\frac{x-4}{(x+3)}=\frac{5}{x(x+3)}$	Eliminate y	M1
	$x\left(x-4\right)=5$	M1: Obtains quadratic equation A1: Correct quadratic equation	M1A1
	$x^2-4x-5=0\Rightarrow x=-1,5$	M1: Solves their quadratic equation as far as $x=\ldots$ A1: Correct critical values	M1A1
	$-3<x\le -1,\ \ 0<x\le 5$ or e.g. $(-3,-1]\cup(0,\ 5]$	M1: Attempts two inequalities using their 4 critical values in ascending order. E.g. $a * x * b,\ c * x * d$ where * is "<" or "≤" and $a<b<c<d$ or equivalent inequalities. Dependent on at least one earlier M mark. A1: All 4 cv's in the inequalities correct A1: Both intervals fully correct	M1A1A1
If the candidate takes the above approach and there is no sketch e.g. just cross multiplies to obtain the critical values -1 and 5 then no marks are available i.e. the cv's 0 and -3 must be stated somewhere to give access to subsequent marks in this case.			
	Considers Regions:		
Way 4	Considers $x<-3\Rightarrow x\left(x+3\right)>0$ $x\left(x-4\right)\le 5\Rightarrow -1\le x\le 5$ But $x<-3$ so no solution	Can be marked as: B1: Critical values 0 and -3 M1: Considers 3 regions M1: Obtains quadratic equation A1: Correct quadratic M1: Solves quadratic A1: cv's -1 and 5 Final 3 marks as already defined.	
	Considers $-3<x<0\Rightarrow x\left(x+3\right)<0$ $x\left(x-4\right)\ge 5\Rightarrow x\ge 5$ or $x\le -1$ But $-3<x<0$ so $-3<x\le -1$		
	Considers $x>0\Rightarrow x\left(x+3\right)>0$ $x\left(x-4\right)\le 5\Rightarrow -1\le x\le 5$ But $x>0$ so $0<x\le 5$		

Question Number	Scheme	Notes	Marks
3.(a)	$r^3-(r-1)^3 \equiv r^3-(r^3-3r^2+3r-1)$ or $r^3-\left(r^3+\binom{3}{1}r^2(-1)+\binom{3}{2}r(-1)^2+(-1)^3\right)$ $\equiv 3r^2-3r+1$ * or $r^3-(r-1)^3 \equiv (r^2+r(r-1)+(r-1)^2)$ $\equiv 3r^2-3r+1$ *	Shows a correct expansion of $(r-1)^3$ or uses $a^3-b^3=(a-b)(a^2+ab+b^2)$ and achieves the printed answer with no errors.	B1*
			(1)
(b)	$n^3-(n-1)^3$ $(n-1)^3-(n-2)^3$ $(n-2)^3-(n-3)^3$ … … 3^3-2^3 2^3-1^3 1^3-0^3	Uses the method of differences. Must include at least $r=1,2,....,n$ or $r=1,....,(n-1),\ n$. But may implied by sight of $\sum r^3-(r-1)^3=n^3$ if insufficient terms shown. If method is clearly other than differences (see note below), then score M0.The final A mark can be witheld if differences not shown i.e. just writes down n^3.	M1
	$n^3=\sum_{r=1}^{n}\left(3r^2-3r+1\right)=\sum_{r=1}^{n}3r^2-\sum_{r=1}^{n}3r+\sum_{r=1}^{n}1$ Sets $n^3=\sum_{r=1}^{n}\left(3r^2-3r+1\right)$ and attempts to expand RHS		M1
	$\sum_{r=1}^{n}1=n$	$\sum_{r=1}^{n}1=n$ seen or implied	B1
	$3\sum_{r=1}^{n}r^2=n(n-1)(n+1)+\frac{3}{2}n(n+1)$	Rearranges to make $k\sum_{r=1}^{n}r^2$ the subject and substitutes for $\sum_{r=1}^{n}r$. **Dependent on the first method mark.**	dM1
	$\sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1)$ **	Completely correct solution with no errors seen.	A1*
	Allow e.g. $\frac{2n^3+3n^2+n}{6}=\frac{1}{6}n(n+1)(2n+1)$		
			(5)
	Note: May be seen in (b): $\sum_{r=1}^{n}r^3-(r-1)^3=\frac{1}{4}n^2(n+1)^2-\frac{1}{4}n^2(n-1)^2=n^3$ etc. Scores a maximum M0M1B1dM0A0 (not using differences)		
	Generally, there are no marks for proof by induction		
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$y = 3\mathrm{e}^{-x}\cos 3x + A\mathrm{e}^{-x}\sin 3x$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -3\mathrm{e}^{-x}\cos 3x - 9\mathrm{e}^{-x}\sin 3x - A\mathrm{e}^{-x}\sin 3x + 3A\mathrm{e}^{-x}\cos 3x$ $\left(= (-3+3A)\mathrm{e}^{-x}\cos 3x + (-9-A)\mathrm{e}^{-x}\sin 3x\right)$ Attempts to differentiate the **given expression** by using the product rule on $3\mathrm{e}^{-x}\cos 3x$ to give $\alpha\mathrm{e}^{-x}\cos 3x + \beta\mathrm{e}^{-x}\sin 3x$ or by using the product rule on $A\mathrm{e}^{-x}\sin 3x$ to give $\alpha A\mathrm{e}^{-x}\cos 3x + \beta A\mathrm{e}^{-x}\sin 3x$		M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (-24-6A)\mathrm{e}^{-x}\cos 3x + (18-8A)\mathrm{e}^{-x}\sin 3x$ (Terms may be uncollected) Uses the product rule again on an expression of the form $\mathrm{e}^{-x}\sin 3x$ or $\mathrm{e}^{-x}\cos 3x$ to give $\alpha\mathrm{e}^{-x}\cos 3x + \beta\mathrm{e}^{-x}\sin 3x$. **Dependent on the first method mark.**		**d**M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 10y = (12-12A)\mathrm{e}^{-x}\cos 3x + (36+4A)\mathrm{e}^{-x}\sin 3x$ Substitute their results into the differential equation. (May be implied)		M1
	$12-12A=0$ or $36+4A=40 \Rightarrow A = ...$	Compares coefficients of $\mathrm{e}^{-x}\sin 3x$ or $\mathrm{e}^{-x}\cos 3x$ and attempts to find A. **Dependent on the previous method mark.**	**d**M1
	$\Rightarrow A = 1$	cao	A1
			(5)
(b) **Marks for (b) can score anywhere in their answer.**	$m^2 - 2m + 10 = 0 \Rightarrow m = 1 \pm 3\mathrm{i}$	M1: Forms and attempts to solve the Auxiliary Equation. See General Principles. A1: Correct solution for the AE	M1 A1
	$(y=)\mathrm{e}^{x}(C\cos 3x + D\sin 3x)$ or $(y=)C\mathrm{e}^{(1+3\mathrm{i})x} + D\mathrm{e}^{(1-3\mathrm{i})x}$	Correct form for CF using their complex roots from the AE	M1
	$y = \mathrm{e}^{x}(C\cos 3x + D\sin 3x) + 3\mathrm{e}^{-x}\cos 3x + \mathrm{e}^{-x}\sin 3x$ GS = their CF + their PI (**Allow ft on their CF and PI**) Must start $y = \ldots$ and depends on at least one the M's being scored and must have been using a PI of the form given.		A1ft
			(4)
(c)	$x=0, y=3 \Rightarrow 3 = C + 3 (\Rightarrow C = 0)$	Attempts to substitute $x = 0$ and $y = 3$ into their answer to (b)	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = (C+3D)\mathrm{e}^{x}\cos 3x + (-3C+D)\mathrm{e}^{x}\sin 3x - 10\mathrm{e}^{-x}\sin 3x$ Attempt to differentiate their GS with or without their C		M1
	$3 = C + 3D$	Attempt to substitute $x = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$y = \mathrm{e}^{x}\sin 3x + 3\mathrm{e}^{-x}\cos 3x + \mathrm{e}^{-x}\sin 3x$	Correct answer. **Must start $y = \ldots$**	A1cao
			(4)
			Total 13

Question Number	Scheme	Notes	Marks
5	$y = e^{\cos^2 x}$		
(a)	$\frac{dy}{dx} = -2\sin x\cos x e^{\cos^2 x} = -\sin 2x e^{\cos^2 x}$ M1: Differentiates using the chain rule to obtain an expression of the form $\alpha \sin x \cos x e^{\cos^2 x}$ or $\beta \sin 2x e^{\cos^2 x}$ A1: Correct derivative Note that candidates may use $\frac{1}{2}(1+\cos 2x)$ instead of $\cos^2 x$		M1A1
	$\frac{d^2y}{dx^2} = -\sin 2x(-2\sin x\cos x e^{\cos^2 x}) - 2\cos 2x e^{\cos^2 x}$ Correct use of the Product Rule on their first derivative **Dependent on the first method mark.**		**d**M1
	$\frac{d^2y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$ *	Achieves the printed answer with no errors.	A1*
	Alternative using logs: $y = e^{\cos^2 x} \Rightarrow \ln y = \cos^2 x \Rightarrow \frac{1}{y}\frac{dy}{dx} = -2\sin x\cos x$ M1: $\frac{1}{y}\frac{dy}{dx} = k\sin x\cos x$ or $k\sin 2x$ A1: $\frac{1}{y}\frac{dy}{dx} = -2\sin x\cos x$ $\frac{dy}{dx} = -y\sin 2x \Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx}\sin 2x - 2y\cos 2x$ M1: Correct use of product rule $\frac{d^2y}{dx^2} = e^{\cos^2 x}(\sin^2 2x - 2\cos 2x)$ * A1: Achieves the printed answer with no errors.		
			(4)
(b)	$\frac{dy}{dx} = 0,\ \frac{d^2y}{dx^2} = -2e$	Both seen, can be implied by subsequent work.	B1
	$f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + ...$ $= e^{\cos^2 0} - \sin 0 e^{\cos^2 0}x + \frac{1}{2}e^{\cos^2 0}(\sin^2 0 - 2\cos 0)x^2 + ...$ Applies the **correct** Maclaurin expansion, the "$\frac{1}{2}$" is required and there must be no x's in the derivatives. This can be implied by their expansion but if the expansion in incorrect for their values and the formula is not quoted, score M0.		M1
	$\left(e^{\cos^2 x}\right) = e(1 - x^2 + ...)$	Or exact equivalent e.g. $e - ex^2$ (i.e. all trig. evaluated)	A1
			(3)
			Total 7

Question Number	Scheme	Notes	Marks
6.	$\cos x\dfrac{dy}{dx}+y\sin x=(\cos^2 x)\ln x$		
	$\dfrac{dy}{dx}+y\dfrac{\sin x}{\cos x}=\cos x\ln x$	Attempt to divide through by $\cos x$. If the intention is not clear, must see at least 2 terms divided by $\cos x$.	M1
	$I=e^{\int\frac{\sin x}{\cos x}dx}=e^{-\ln\cos x}$	M1: $e^{\int \pm\text{their } P(x)\,(dx)}$. Dependent on the first method mark. A1: $e^{-\ln\cos x}$ or $e^{\ln\sec x}$	dM1A1
	$=\dfrac{1}{\cos x}$	$\dfrac{1}{\cos x}$ or $(\cos x)^{-1}$ or $\sec x$	A1
	$\dfrac{y}{\cos x}=\int\ln x\,dx$ or $\dfrac{d}{dx}\left(\dfrac{y}{\cos x}\right)=\ln x$	M1: $y\times\text{their } I=\int Q(x)\times\text{their } I\,dx$ or $\dfrac{d}{dx}(y\times\text{their } I)=Q(x)\times\text{their } I$ A1: $\dfrac{y}{\cos x}=\int\ln x\,dx$ or $\dfrac{d}{dx}\left(\dfrac{y}{\cos x}\right)=\ln x$	M1A1
	$\dfrac{y}{\cos x}=x\ln x-x+C$	Attempts $\int\ln x\,dx$ by parts correctly (correct sign needed unless correct formula quoted and used).	M1
	$y=(x\ln x-x+C)\cos x$	Any equivalent with the constant correctly placed and "$y=\ldots$" must appear at some stage.	A1
			Total 8
	Note: Failure to divide by cos x at the start would mean that only the 3rd Method mark is available.		

Question Number	Scheme	Notes	Marks
7			
(a)	$y = r\sin\theta = 4\cos 2\theta \sin\theta$	Attempts to use $r\sin\theta$	M1
	$\frac{dy}{d\theta} = 4\cos 2\theta\cos\theta - 8\sin 2\theta\sin\theta$ or $y = 4\left(1-2\sin^2\theta\right)\sin\theta = 4\sin\theta - 8\sin^3\theta \Rightarrow \frac{dy}{d\theta} = 4\cos\theta - 24\sin^2\theta\cos\theta$ A correct expression for $\frac{dy}{d\theta}$ **or any multiple of** $\frac{dy}{d\theta}$		B1
	$\frac{dy}{d\theta} = 0 \Rightarrow \theta = ...$	Set their $\frac{dy}{d\theta} = 0$ and attempt to solve to obtain a value for θ	M1
	$r = \frac{8}{3},\ \theta = 0.421,\ \theta = 2.72$	Any one of: $r = \frac{8}{3}$ (or awrt 2.7) or $\theta = 0.421...$ or $\theta = 2.72...$	A1
	$r = \frac{8}{3}$ $\theta = 0.421,\ 2.72$	Correct value for *r* and both angles correct. May be seen as $\left(\frac{8}{3}, 0.421\right), \left(\frac{8}{3}, 2.72\right)$. Allow $\left(0.421, \frac{8}{3}\right), \left(2.72, \frac{8}{3}\right)$ but coordinates do not have to be paired and accept awrt 0.421, 2.72 and allow awrt 2.7 for $\frac{8}{3}$. Ignore any other coordinates given once the correct values have been seen.	A1
			(5)

(b)	$A = \ldots\int (4\cos 2\theta)^2\,\mathrm{d}\theta$	Indication that the integration of $(4\cos 2\theta)^2$ is required. Ignore any limits and ignore any constant factors at this stage.	M1
	$\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)$	A correct identity seen or implied.	A1
	$A = \ldots[\alpha\theta + \beta\sin 4\theta]$	Integrates to obtain an expression of the form $\alpha\theta + \beta\sin 4\theta$. Ignore any limits and ignore any constant factors. Dependent on the first method mark.	**d**M1
	$= 16\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{4}}$	A fully correct method that if evaluated correctly would give the answer 4π. Note that the correct "constant factor" may only be applied at the very last stage of their working and this method mark would only be awarded at that point. **Dependent on all previous method marks.**	**dd**M1
	Examples that could score the final M1 (following correct work): $16\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{4}}$, $8\left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$, $8\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$, $16\left[\theta + \frac{1}{4}\sin 4\theta\right]_{\frac{3\pi}{4}}^{\pi}$		
	$= 4\pi$	cao	A1
			(5)

(c)	$PQ = 2r\sin\theta = \frac{16}{3\sqrt{6}}$	Correct expression or value for PQ or $PQ/2$. E.g. $2\left(\frac{8}{3}\right)\frac{1}{\sqrt{6}}$, $2\left(\frac{8}{3}\right)\sin 0.421$, $2\left(\frac{8}{3}\right)\sin 2.72$, $\frac{8\sqrt{6}}{9}$ or half of these. May be implied by awrt 2.2 or awrt 1.1	B1
	$SP = 8$ or $\frac{SP}{2} = 4$	Correct value for SP or $SP/2$	B1
	Area $PQRS = \frac{16}{3\sqrt{6}} \times 8 \left(= \frac{64\sqrt{6}}{9}\right)$	Their $PQ \times SP$. Must be the complete rectangle here.	M1
	Required area $= \frac{128}{3\sqrt{6}} - 4\pi$	M1: Their rectangle area – their answer to part (b) A1: Correct exact answer or equivalent exact form e.g. $\frac{64\sqrt{6}}{9} - 4\pi$ or allow awrt 4.8 or 4.9	M1A1
			(5)
			Total 15

Question Number	Scheme	Notes	Marks
8(a)(i)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (c+\mathrm{i}s)^5 = c^5 + 5c^4\mathrm{i}s + 10c^3\mathrm{i}^2s^2 + 10c^2\mathrm{i}^3s^3 + 5c\mathrm{i}^4s^4 + \mathrm{i}^5s^5$ Attempts to expand $(c+\mathrm{i}s)^5$ including binomial coefficients (NB may only see real terms here)		M1
	$\cos 5\theta = \mathrm{Re}(c+\mathrm{i}s)^5 = c^5 + 10c^3\mathrm{i}^2s^2 + 5c\mathrm{i}^4s^4 = c^5 - 10c^3s^2 + 5cs^4$ Extracts real terms and uses $\mathrm{i}^2 = -1$ to eliminate i.		M1
	$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$* Achieves the printed result with no errors seen.		A1*
	Alternative: $\left(z = \cos\theta + \mathrm{i}\sin\theta,\ z^{-1} = \cos\theta - \mathrm{i}\sin\theta,\ z^n = \cos n\theta + \mathrm{i}\sin n\theta\right)$ $\left(z+\frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z^2}\right) + 10z^2\left(\frac{1}{z^3}\right) + 5z\left(\frac{1}{z^4}\right) + \frac{1}{z^5}$ $(2\cos\theta)^5 = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ **M1:** Expands $\left(z+\frac{1}{z}\right)^5$ including binomial coefficients and uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ at least once to obtain an equation in cos $\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta \Rightarrow \cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 15\cos\theta - 10\cos\theta$ **M1:** Uses **correct** identity for cos3θ to obtain $\cos 5\theta$ in terms of single angles $= \cos^5\theta + 15\cos\theta\left(1-\sin^2\theta\right)^2 - 20\cos^3\theta + 5\cos\theta$ $= \cos^5\theta - 10\cos\theta\sin^2\theta + 15\cos\theta\sin^4\theta$ $= \cos^5\theta + 5\cos\theta\sin^4\theta + 10\cos\theta\sin^2\theta\left(\sin^2\theta - 1\right)$ $\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$* **A1:** Achieves the printed result with no errors seen (may need careful checking)		
(ii)	$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ This expression (or equivalent) with no i's seen. **Note that some candidates may re-start and expand here as above.**		B1
			(4)
(b)	$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$ Uses $\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta}$ and substitutes the results from part (a)		M1
	$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta} = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$* Achieves the printed result with no errors seen.		A1*
	Note that a minimum could be: $\tan 5\theta = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$* **Note that some candidates may work backwards which is acceptable:** E.g. $\frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1} = \frac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1}$ $= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} = \tan 5\theta$		
			(2)

(c)	$\tan 5\theta = 0$ or $\dfrac{\tan^5\theta - 10\tan^3\theta + 5\tan\theta}{5\tan^4\theta - 10\tan^2\theta + 1} = 0$ or $\dfrac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1} = 0$	Considers $\tan 5\theta = 0$. This may be implied by $\tan^5\theta - 10\tan^3\theta + 5\tan\theta = 0$ or $t^5 - 10t^3 + 5t = 0$ or $\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	M1
	$\tan^5\theta - 10\tan^3\theta + 5\tan\theta = 0$ or $t^5 - 10t^3 + 5t = 0$	Equate numerator to 0 This may be implied by $\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	M1
	$\tan^4\theta - 10\tan^2\theta + 5 = 0$ or $t^4 - 10t^2 + 5 = 0$	Correct quartic	A1
	$x^2 - 10x + 5 = 0$	$x^2 - 10x + 5 = 0$ or equivalent	A1
			(4)
(d)	Product of roots: $\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5$ Or solves "$x^2 - 10x + 5 = 0$" and attempts to multiply roots together e.g. $x = \dfrac{10 \pm \sqrt{100-20}}{2} = 5 \pm 2\sqrt{5}$ and $\left(5+2\sqrt{5}\right)\left(5-2\sqrt{5}\right) = \ldots$	Must clearly state product of roots or e.g. $\alpha\beta = 5$ or $x_1x_2 = 5$ and uses their constant in (c) or solves their quadratic and attempts product of roots.	M1
	$\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5 \Rightarrow \tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = \sqrt{5}$ *	Shows the given result with no errors.	A1
			(2)
			Total 12